JEE MAIN 2020 3rd SEPT SHIFT 2

Question 1. Dimension of solar constant is:
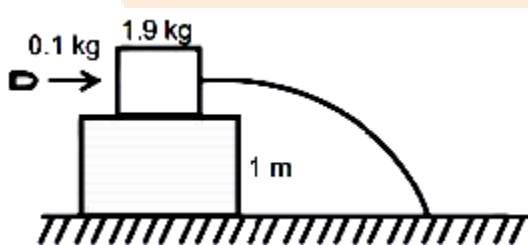
    (1) $M^1L^0T^{-3}$

    (2) $M^1L^1T^{-3}$

    (3) $M^0L^0T^3$

    (4) $M^1L^2T^{-3}$

Ans. (1)

Sol. Solar constant $= \dfrac{\text{Energy}}{\text{Time Area}}$

$= \dfrac{M^1L^2T^{-2}}{TL^2} = M^1L^0T^{-3}$

Question 2. In the given diagram a 0.1 kg bullet moving with speed 20 m/sec strikes 1.9 kg mass and get embedded in it. Find the kinetic energy of the mass with which it will strikes the ground is.

    (1) 11 J

    (2) 21 J

    (3) 25 J

(4) 30 J

Ans. (2)

Sol. Conservation of linear momentum

$0.1 \times 20 = (0.1 + 1.9) \times v$

$v = 1 \text{ m/s}$

Using work energy theorem

$W_g = \Delta k$

$2 \times g \times 1 = k - \dfrac{1}{2} \times 2 \times 1^2$

$\therefore k = 21J$

Question 3. A mass m is moving in SHM on a line with amplitude A and frequency f in a spring-mass system. Suddenly half of the mass comes to rest just at the moment, when it crosses mean position, then the new amplitude becomes λA, then λ will be

(1) $\dfrac{1}{2}$

(2) $\dfrac{1}{\sqrt{2}}$

(3) $\sqrt{2}$

(4) 1

Ans. (3)

Sol. As the external force is absent, by conservation of linear momentum

$$m_l u_i = m_f v_f$$

$$m.A\omega = \frac{m}{2}A'\omega'$$

$$m.A\sqrt{\frac{k}{m}} = \frac{m}{2}A'\sqrt{\frac{k}{m/2}}$$

$$A = \frac{1}{2}\sqrt{2}A'$$

$$A' = \sqrt{2}A$$

Question 4. A loop of area 'S' m^2 and N turns carrying current 'i' is placed in a uniform magnetic field 'B' with its plane parallel to $\vec{B}$. If torque 'τ' is experience by loop due to magnetic field, find $|\vec{B}|$
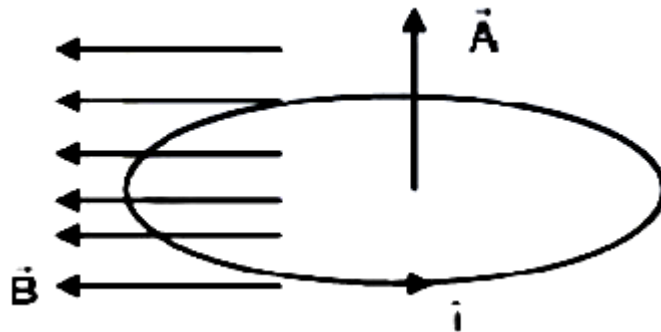
(1) $\dfrac{\tau}{NiS}$

(2) $\dfrac{N\tau}{iS}$

(3) $\dfrac{i\tau}{NS}$

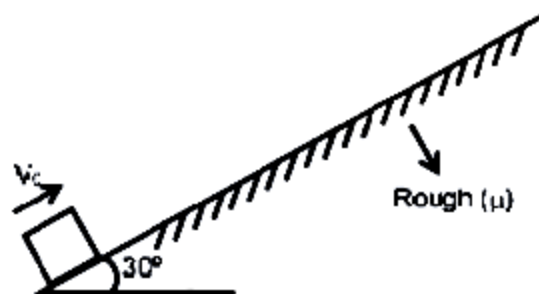(4) $\dfrac{S\tau}{Ni}$

Ans. (1)

Sol.

$$\tau = \left|\vec{M} \times \vec{B}\right| = NiAB\sin(90°)$$

$$= NiAB = NiSB$$

$$\Rightarrow B = \frac{\tau}{NiS}$$

Question 5. A block starts going up a rough inclined plane with speed V_0 as shown in figure. After some time reaches to starting point again, with a speed $\frac{V_0}{2}$. Find coefficient of friction 'μ' . Given g = 10 m/s2.
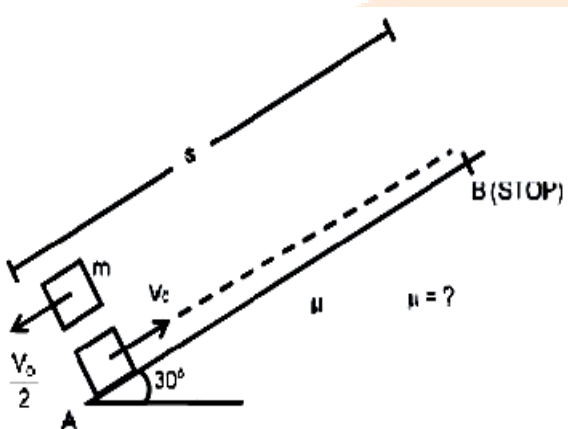

(1) 0.15

(2) 0.35

(3) 0.75

(4) 0.80

Ans. (2)

Sol.

A to B

$a_1 = g \sin 30° + \mu g \cos 30°$

$= \dfrac{g}{2} + \dfrac{\mu g \sqrt{3}}{2}$; $g = 10 \text{ m/s}^2$

$V_0^2 - 2a_1(s) = 0$

$s = \dfrac{V_0^2}{2a_1}$ ... (i)

B to A

$a_2 = \dfrac{g}{2} - \dfrac{\mu \sqrt{3}}{2}g$

$\left(\dfrac{V_0}{2}\right)^2 = 2a_2(s)$

$$s = \frac{V_0^2}{8a_2} \qquad \ldots \text{(ii)}$$

From equation (i) and (ii)

$$\frac{V_0^2}{a_1} = \frac{V_0^2}{4a_2}$$

$$\Rightarrow a_1 = 4a_2$$

$$\Rightarrow 5 + 5\sqrt{3}\mu = 4\left(5 - 5\sqrt{3}\mu\right)$$

$$\Rightarrow 5 + 5\sqrt{3}\mu = 20 - 20\sqrt{3}\mu \Rightarrow 25\sqrt{3}\mu = 15$$

$$\Rightarrow \mu = \frac{\sqrt{3}}{5} = 0.35$$

Question 6. An ideal gas is heated by 160 J at constant pressure, its temperature rises by 50°C and if 240 J of heat is supplied at constant volume, temperature rises by 100°C, then its degree of freedom should be:

(1) 3

(2) 5

(3) 6

(4) 7

Ans. (3)

Sol. At constant pressure:

$$\Delta Q = nC_P\Delta T$$

$\Rightarrow 160 = nC_P.50$ ... (1)

At constant volume

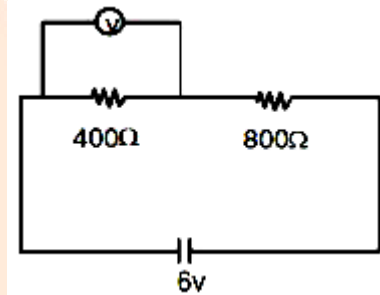$\Delta Q = nC_v \Delta T$

$\Rightarrow 240 = nC_v.100$ ... (2)

Equation (1) divided by (2)

$$\frac{160}{240} = \frac{C_P}{C_v}.\frac{50}{100}$$

$$\frac{C_P}{C_v} = \frac{4}{3} = 1 + \frac{2}{f}$$

$f = 6$

Question 7. In the given diagram resistance of voltmeter is 10 kΩ . Find reading of the voltmeter.
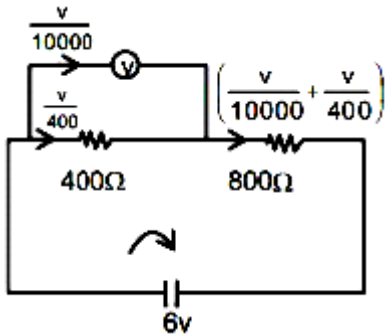

(1) 4v

(2) 3.23v

(3) 1.95v

(4) 1.26v

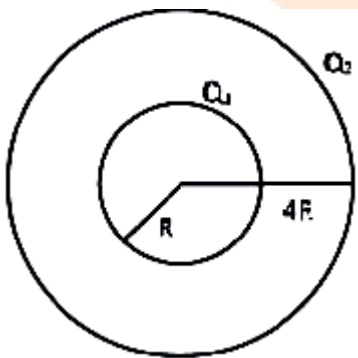Ans. (3)

Sol.

Let voltmeter reading is v

$$\frac{v}{400} \times 400 + \left(\frac{v}{10000} + \frac{v}{400} \right) 800 = 6$$

$$\Rightarrow v + \frac{8v}{100} + 2v - 6$$

$$\frac{77v}{25} = 6$$

$$v = \frac{150}{77} = 1.95v$$

Question 8. In the given figure, there are two concentric spherical shells, find potential difference between the spheres

(1) $\dfrac{3}{8\pi \in_0} \cdot \dfrac{Q_1}{R}$

(2) $\dfrac{3}{16\pi \in_0} \cdot \dfrac{Q_2}{R}$

(3) $\dfrac{3}{4\pi \in_0} \cdot \dfrac{Q_1}{R}$

(4) $\dfrac{3}{16\pi \in_0} \cdot \dfrac{Q_1}{R}$

Ans. (4)

Sol. $V_{inner} = \dfrac{KQ_1}{R} + \dfrac{KQ_2}{4R}$

$V_{outer} = \dfrac{KQ_1}{4R} + \dfrac{KQ_2}{4R}$

Potential difference

$\Delta V = V_{inner} - V_{outer}$

$= \dfrac{3}{4} \cdot \dfrac{KQ_1}{R} = \dfrac{3}{16\pi \in_0} \cdot \dfrac{Q_1}{R}$

Question 9. A body cools from 50°C to 40°C in 5 minutes in surrounding temperature 20°C. Find temperature of body in next 5 minutes.

(1) 13.3°C

(2) 23.3°C

(3) 43.3°C

(4) 33.3°C

Ans. (4)

Sol. Using Newton's Law of cooling

$$\frac{50 - 40}{5 \text{Min}} = K\left(\frac{50 + 40}{2} - 20\right) \qquad \text{...... (i)}$$

Next 5 Min.

$$\frac{40 - \theta}{5} = K\left(\frac{40 + \theta}{2} - 20\right) \qquad \text{...... (ii)}$$

Dividing (ii) / (i)

$$\frac{40 - \theta}{10} = \frac{40 + \theta - 40}{50 + 40 - 40} = \frac{\theta}{50}$$

$$40 - \theta = \frac{\theta}{5}$$

$$200 - 5\theta = \theta$$

$$\therefore \theta = \frac{200}{6} = 33.3°C$$

Question 10. A square wire loop of side 30 cm & wire cross section having diameter 4 mm is placed perpendicular to a magnetic field changing at the rate 0.2 T/s. Find induced current in the wire loop.

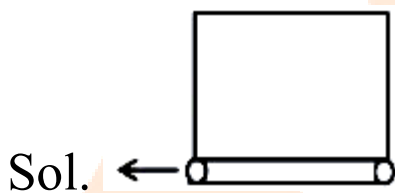(Given: Resistivity of wire material is 1.23×10^{-8} Ωm)

(1) 5.34×10^2 A

(2) 3.34×10^2 A

(3) 7.34×10^2 A

(4) 1.34×10^2 A

Ans. (1)

Sol.

$$R_{wire} = \frac{\rho l}{A}$$

$$\phi = BA; \; |e| = \frac{d\phi}{dt} = \frac{dB}{dt}(A)$$

$$i = \frac{e}{R} = \left|\frac{dB}{dt}\right| \frac{(A)^2}{\rho \times L} = \frac{0.2 \times \left\{\pi \times 2 \times 10^{-3}\right\}^2}{1.23 \times 10^{-8} \times 4 \times 0.3}$$

$$= \frac{0.2 \times \pi^2 \times 10^{-6}}{1.23 \times 10^{-8} \times 0.3}$$

$$= 5.34 \times 10^2 \, A$$

Question 11. Electric field of an electromagnetic wave is $\vec{E} = E_0 \cos\left(\omega t - kx\right)\hat{j}$. The equation of corresponding magnetic field at t = 0 should be:

(1) $\vec{B} = E_0 \sqrt{\mu_p \in_0} \cos k \left(\hat{k}\right)$

(2) $\vec{B} = \dfrac{E_0}{\sqrt{\mu_0 \in_0}} \cos k \times \hat{k}$

(3) $\vec{B} = E_0 \sqrt{\mu_0 \in_0} \cos kx \left(-\hat{k}\right)$

(4) $\vec{B} = \dfrac{E_0}{\sqrt{\mu_0 \in_0}} \cos k \times \left(-\hat{k}\right)$

Ans. (1)

Sol. $B_0 = \dfrac{E_0}{C} = \dfrac{E_0}{1/\sqrt{\mu_0 \in_0}} = E_0 \sqrt{\mu_0 \in_0}$

As the light is propagating in x direction

& $\hat{E} \times \hat{B} \parallel \hat{C}$

$\therefore \hat{B}$ should be in $\hat{k}$ direction

Hence, option (1) is correct.

Question 12. Given two points sources having same power of 200W. One source is emitting photons of $\lambda_1 = 500$ nm and other emitting X-ray photons of $\lambda_2 = 1$ nm . Find ratio of photon density from both the sources?

(1) 200

(2) 500

(3) 250

(4) 0.4

Ans. (2)

Sol. Ps - Power of source

$$P_s = n\frac{hc}{\lambda}; \ n = \text{no. of photons emitted/s}$$

$$\Rightarrow n \propto \lambda \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = 500$$

Question 13. Image of a point object placed on principal axis at a distance of 30 cm from the mirror is formed at 10 cm from the mirror. If object start moving with velocity 9 cm/sec. Find initial velocity of image.

(1) -9 cm

(2) -4 cm
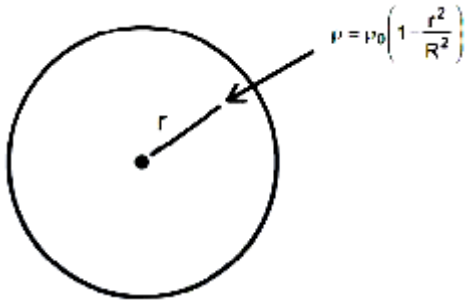
(3) -1 cm

(4) -3 cm

Ans. (1)

Sol. Velocity of image

$$v_i = -\frac{v^2}{u^2} \times v_0$$

$$= -\frac{10^2}{30^2}(9)$$

$$= -1 \text{ cm/sec}$$

Question 14. Mass density of a sphere having radius R varies as $\rho = \rho_0 \left(1 - \dfrac{r^2}{R^2}\right)$. Find maximum magnitude of gravitational field.
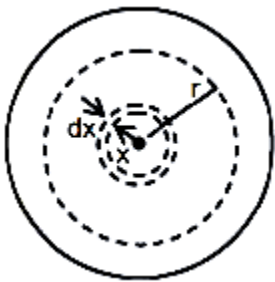
(1) $\dfrac{4}{3}\pi G \rho_0 R$

(2) $\dfrac{2\sqrt{3}}{5}\pi G \rho_0 R$

(3) $\dfrac{8\sqrt{5}}{27}\pi G \rho_0 R$

(4) $\dfrac{2\sqrt{5}}{27}\pi G \rho_0 R$

Ans. (3)

Sol.

$$dm = \rho \times 4\pi x^2 dx$$

$$= \rho_0 \left(1 - \frac{x^2}{r^2} \right) \times 4\pi x^2 dx$$

$$m = 4\pi\rho_0 \int_0^r \left(x^2 - \frac{x^4}{R^2} \right) dx$$

$$m = 4\pi\rho_0 \left| \frac{r^3}{3} - \frac{r^5}{5R^2} \right|$$

$$E = \frac{Gm}{r^2}$$

$$= \frac{G}{r^2} \times 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2} \right)$$

$$E = 4\pi G\rho_0 \left(\frac{r}{3} - \frac{r^3}{5R^2} \right)$$

E is maximum when

$$\frac{dE}{dr} = 0 \Rightarrow \frac{dE}{dr} = 4\pi G\rho_0 \left(\frac{1}{3} - \frac{3r^2}{5R^2} \right) = 0$$

$$\Rightarrow r = \frac{\sqrt{5}}{3} R$$

$$E_{max} = 4\pi G\rho_0 \times \frac{\sqrt{5}R}{3} \left[\frac{1}{3} - \frac{1}{5} \times \frac{5}{9} \right]$$

$$E_{max} = \frac{8\sqrt{5}}{27} \pi G\rho_0 R$$

Question 15. Two light rays having the same wavelength λ in vacuum are in phase initially. Then the first ray travels a path L_1 through a medium of refractive index n1 while the second ray travels a path of length L_2 through a medium of refractive index n_2. The two waves are then combined to produce interference. The phase difference between the two waves at the point of interference is:

(1) $\dfrac{2\pi}{\lambda}\left(L_2 - L_1\right)$

(2) $\dfrac{2\pi}{\lambda}\left(n_1 L_1 - n_2 L_2\right)$

(3) $\dfrac{2\pi}{\lambda}\left(n_2 L_1 - n_1 L_2\right)$

(4) $\dfrac{2\pi}{\lambda}\left(\dfrac{L_1}{n_1} - \dfrac{L_2}{n_2}\right)$

Ans. (2)

Sol. The optical path between any two points is proportional to the time of travel. The distance traveled by light in a medium of refractive index μ in time t is given by

$d = vt$ ... (i)

where v is velocity of light in the medium. The distance traversed by light in a vacuum in this time, $\Delta = ct$

$= c \times \dfrac{d}{v}$ [from equation (i)]

$$= d\frac{c}{v} = \mu d \quad \text{...... (ii)} \qquad \left(\text{Since, } \mu = \frac{c}{v}\right)$$

This distance is the equivalent distance in vacuum and is called optical path.

Here, optical path for first ray $= n_1L_1$

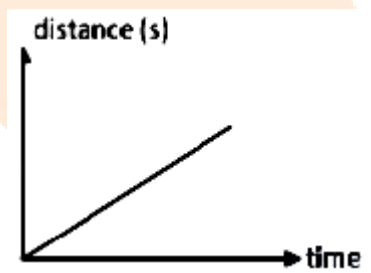Optical path for second ray $= n_2L_2$

Path difference $= n_1L_1 - n_2L_2$

Now phase difference
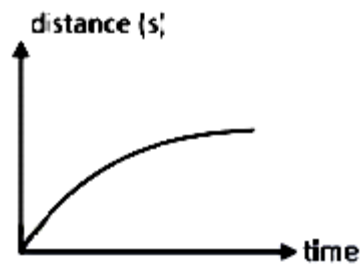
$$= \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} \times (n_1L_1 - n_2L_2)$$

Question 16. Constant power P is supplied to particle having mass m, initially at rest. Choose correct graph.

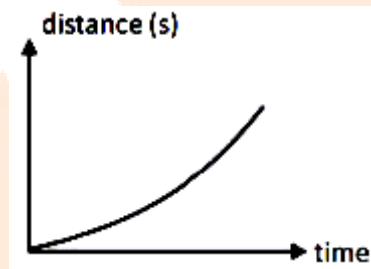(1)

(2)

(3)

(4)

Ans. (3)

Sol. $P.t = \dfrac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\dfrac{2P}{m}}\right)t^{1/2}$

$s = \int\limits_0^t vdt = \sqrt{\dfrac{2P}{m}}\int\limits_0^t t^{1/2}dt$

$= \sqrt{\dfrac{2P}{m}} \cdot \dfrac{t^{3/2}}{3/2}$

$s = \sqrt{\dfrac{8P}{9m}} \cdot t^{3/2}$

Question 17. A p - n junction becomes active as photons of wavelength; $\lambda = 400$ nm falls on it. Find the energy band gap?
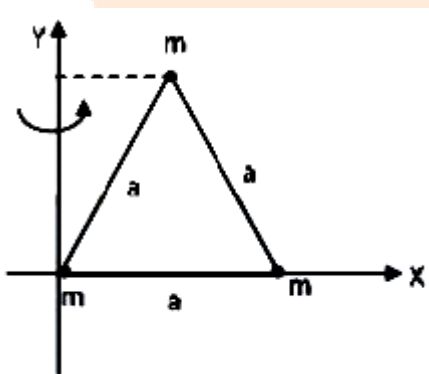
(1) 3.09 eV

(2) 4.51 eV

(3) 2.45 eV

(4) 5.34 eV

Ans. (1)

Sol. $\lambda = 400$ nm

Band gap $E_g = \dfrac{hc}{\lambda} = \dfrac{1237.5}{400} = 3.09$ eV

Question 18. In the diagram three point masses 'm' each are fixed at the corners of an equilateral triangle. Moment of inertia of the system about y-axis is $\dfrac{N}{20}$ ma², N is:
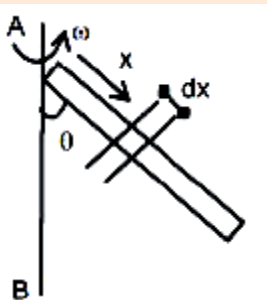

(1) 25

(2) 50

(3) 75

(4) 100

Ans. (1)

Sol. $I = m \times O^2 + ma^2 + m\left(\dfrac{a}{2}\right)^2$

$= \dfrac{5}{4}ma^2 = \dfrac{25}{20}ma^2$

$N = 25$

Question 19. A rod is rotating with angular velocity ω about axis AB. Find $\cos \theta$

(1) $\dfrac{g}{2\ell\omega^2}$

(2) $\dfrac{g}{\ell\omega^2}$

(3) $\dfrac{2g}{\ell\omega^2}$

(4) $\dfrac{3g}{2\ell\omega^2}$

Ans. (4)

Sol. Torque of centrifugal force

$$\tau_{cf} = dm.x \sin\theta\omega^2 x \cos\theta = \frac{m}{\ell}\omega^2 \sin\theta \cos\theta \int_0^\ell x^2 dx$$
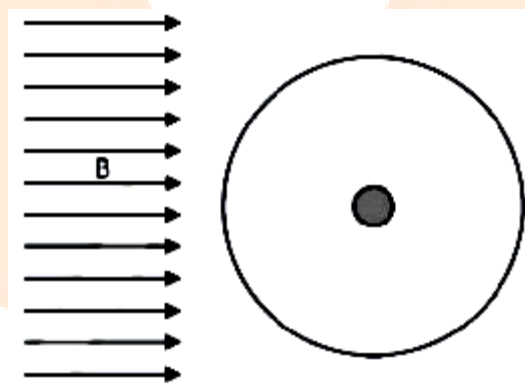
$$\tau_{ef} = \frac{m\ell^2 \omega^2 \sin\theta \cos\theta}{3}$$

$$\tau_{mg} = \tau_{cf}$$

$$mg\frac{\ell}{2}\sin\theta = \frac{m\ell^2 \omega^2 \sin\theta \cos\theta}{3}$$

$$\cos\theta = \frac{3g}{2\ell\omega^2}$$

Question 20. In a diamagnetic sphere a cavity is made at its centre and now paramagnetic material is inserted in the cavity. The sphere is kept in a external magnetic field at centre.

(1) 0

(2) B

(3) $B0 > B$

(4) $B0 < B$

Ans. (1)

Sol. When magnetic field is applied diamagnetic substance produces magnetic field in opposite direction so net magnetic field will be zero.

Chemistry

Question 21. A mixture of one mole of each of $O_2(g)$ $H_2(g)$, He(g) exists in a container of volume V at temperature T, in which partial pressure of $H_2(g)$ is 2 atm. The total pressure in the container is:

(1) 6 atm

(2) 18 atm

(3) 33 atm

(4) 24 atm

Answer: (1)

Solution:

$$\boxed{P_{gas} = \frac{n_{gas} RT}{V}}$$

as n, t & V constant So

$$P_{H_2} = P_{O_2} = P_{He} = 2\,atm$$

So, $P_{Total} = P_{H_2} + P_{O_2} + P_{He}$

$= 6 atm$

Question 22. What is the concentration and %(w/w) of 5.6 V H_2O_2 solution? [Given molar mass of H_2O_2 = 34 g/mol density = 1 g/mL]

(1) 0.5, 1.70

(2) 0.25, 1.70

(3) 0.5, 0.85

(4) 0.25, 0.85

Answer: (1)

Solution:

For H_2O_2

$$Molarity = \frac{Volume \ strength}{11.2} = \frac{5.6}{11.2} = 0.5 \ M$$

$$Molarity = \frac{\%(w/w) \times 10 \times d}{GMM}$$

$$0.5 = \frac{\%(w/w) \times 10 \times d}{34}$$

$$\%(w/w) = \frac{0.5 \times 34}{10} = 1.7$$

Question 23. Find incorrect statement about manganate and permanganate ions.

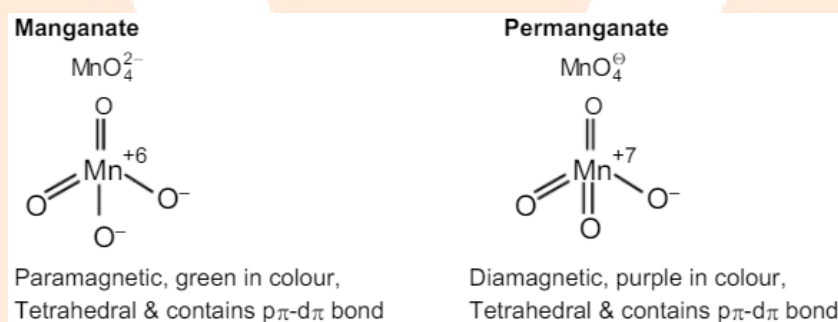(1) Both manganate and permanganate ions are Paramangnetic

(2) Manganate ion is green in colour while parmanganate ion is purple in colour

(3) Both manganate and permanganate ions have tetrahedral shape

(4) In manganate and permanganate ions Mn from pπ-pπ bond with oxygen.

Answer: (1)

Solution:

Manganate MnO_4^{2-}

Paramagnetic, green in colour, Tetrahedral & contains pπ-dπ bond

Permanganate $MnO_4^{\ominus}$

Diamagnetic, purple in colour, Tetrahedral & contains pπ-dπ bond

Question 24. Among the following statements identify the correct set of statements

(a) Size of Be is smaller than Mg

(b) Ionisation energy of Be is greater than Al

(c) Both Be and Al form covalent compounds readily

(d) Both Be and Al does not react with nitrogen
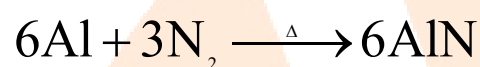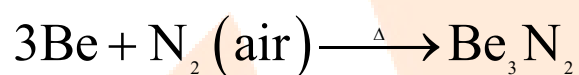
   (1) a, b, c

   (2) a, c, d

(3) b, c, d

(4) a, b, d

Answer: (1)

Solution:

Both Be and Al react with nitrogen to form nitride

$$3Be + N_2 \, (air) \xrightarrow{\Delta} Be_3N_2$$
$$6Al + 3N_2 \xrightarrow{\Delta} 6AlN$$
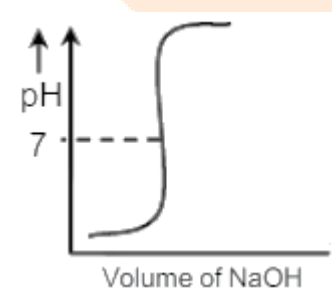
Remaining all statements are correct.

Question 25. In 0.1 M HCl solution, 0.1 M NaOH solution is added gradually then identify the correct graph this titration.

(1)

(2)

(3)

Volume of NaOH

(4)

Volume of NaOH
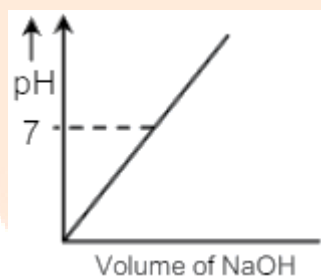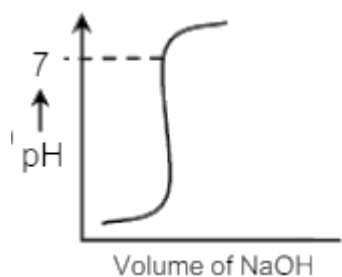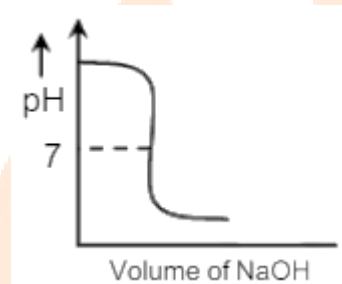
Answer: (2)

Solution:

At equivalence point pH is 7 and pH increases with addition of NaOH so correct graph is (b).

Question 26. For the reaction, $2A + 3B + \dfrac{3}{2}C \rightarrow 3P$

The correct relation between the rate of reaction of species A, B and C is:

(1) $\dfrac{dn_A}{dt} = \dfrac{2}{3}\dfrac{dn_B}{dt} = \dfrac{4}{3}\dfrac{dn_C}{dt}$

(2) $2\dfrac{dn_A}{dt} = 3\dfrac{dn_B}{dt} = \dfrac{3}{2}\dfrac{dn_C}{dt}$

$$(3) \quad \frac{3}{2} \frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{3}{4} \frac{dn_C}{dt}$$

$$(4) \quad \frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$$

Answer: (1)

Solution:

For a given reaction

$$\text{rate} = -\frac{1}{2} \frac{dn_A}{dt} = -\frac{1}{3} \frac{dn_B}{dt} = -\frac{2}{3} \frac{dn_C}{dt}$$

$$\text{rate} = \frac{dn_A}{dt} = \frac{2}{3} \frac{dn_B}{dt} = \frac{4}{3} \frac{dn_C}{dt}$$

Question 27. The crystal field configuration of complexes $[Ru(en)_3]Cl_2$ and $[Fe(H_2O)_6]^{2+}$ respectively is:

(1) t_2g_4, eg_2 and t_2g_6, eg_0

(2) t_2g_6, eg_0 and t_2g_4, eg_0

(3) t_2g_4, eg_2 and t_2g_4, eg_2

(4) t_2g_6, eg_0 and t_2g_6, eg_0

Answer: (2)

Solution:

$[Ru(en)_3]Cl_2 \Rightarrow Ru^{2+} = 4d_6 = t_2g_6, 6g_0$

$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} = 3d_6 \, t_2g_4, eg_2$
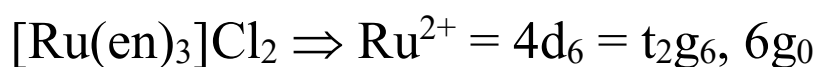
So, correct answer is (2).

Question 28. What is the valency of an atom if its successive ionisation energies respectively are 800, 25356, 32456 kJ/mole?

(1) 3

(2) 4

(3) 5

(4) 6

Answer: (1)

Solution:

As difference in 3rd and 4th ionisation energies is high so atom contains 3 valence electrons. Hence valency of the atom is 3.

Question 29. For a hypothetical case let value of l is define as 0, 1, 2, 3 …..(n + 1) for principle quantum number n

(1) Atomic number of 1st noble gas is 8

(2) Atomic number of 1st alkali metal is 9

(3) Carbon has electron in 2pz

(4) Element with atomic number 13 has half-filled valence shell

Answer: (4)

Solution:

For n = 1   valence of l = 0, 1, 2   Electronic configuration = 1s21p61d10

(1) 1st noble gas atomic number is 18

(2) 1st alkali metal electronic configuration $\Rightarrow$ 1s21p61d102s1 $\Rightarrow$ (Z = 19)

(3) Electronic configuration of C(Z = 6) $\Rightarrow$ 1s21p4

(4) Z = 13, Electronic configuration = 1s21p61d5

So it has half-filled electronic configuration.

Question 30. A current of 2A is passed through a dichromate solution for 5 min. Then 0.104 g of Cr^{3+} ions are formed. What is the percentage efficiency of cell?

[Given $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$, Atomic mass of Cr = 52]

Answer: 96.50

Solution:

Charge (q) = it = 2 × 5 × 60

$$= 600C = \left(\frac{600}{96500}F\right)$$

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \quad \rightarrow \quad 2Cr^{3+} \quad + \quad 7H_2O$$
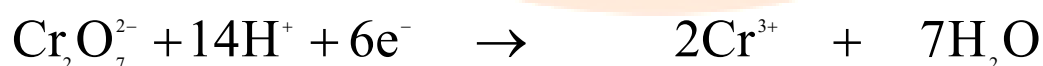
$$\left(\frac{600}{96500}\right)F \quad \frac{1}{3}\left(\frac{600}{96500}Mole\right)$$

Theoretical mass of Cr3+ $= \dfrac{1}{3} \times \dfrac{600}{96500} \times 52\,\text{g}$

So, efficiency $=$

$\dfrac{W_{actual}}{W_{Theoritial}} \times 100 = \dfrac{0.104 \times 3 \times 96500}{52 \times 600} \times 100 = 96.50$

Question 31. How much volume of 0.1 N NaOH will neutralize 10 mL of 0.1 N phosphoric acid?

Answer: 10.00

Solution:

Phosphoric acid is phosphorous acid (H_3PO_3).

$NaOH + H_3PO_3 \rightarrow NaH_2PO_3 + H_2O$

For neutrization

$\left(N_1V_1\right)_{acid} = \left(N_2V_2\right)_{base}$

$0.1 \times 10 = 0.1 \times \left(V_{mL}\right)_{NaOH}$

$V_{NaOH} = 10\,\text{mL}$

Question 32. In Isotonic solution of protein A and protein B, 0.73 gram of protein A is dissolved in 250 ml while 1.65 gram of protein B is dissolved in 1 L solution, then what is the ratio of molecular mass of Protein A and protein B?

Answer: 01.77

Solution:

For isotonic solution

$I_1C_1 = i_2C_2$ {For protein i = 1}

$C_1 = C_2$

$$\Rightarrow \frac{0.73 \times 1000}{M_A \times 250} = \frac{1.65}{M_B \times 1}$$

$$\frac{M_A}{M_B} = \frac{0.73 \times 4}{1.65} = 1.77$$

Question 33. 6.022×10^{22} molecules of a compound X has mass 10 g. What is the molarity of solution containing 5g of 'X' in 2 Lit. solution, answer as P [Where M = P × 10^{-3} Mole/Lit]

Answer: 25.00

Solution:

$$\text{Number of mole of X} = \frac{6.022 \times 10^{22}}{6.022 \times 10^{23}} = \frac{10}{\text{Molar mass of X}}$$
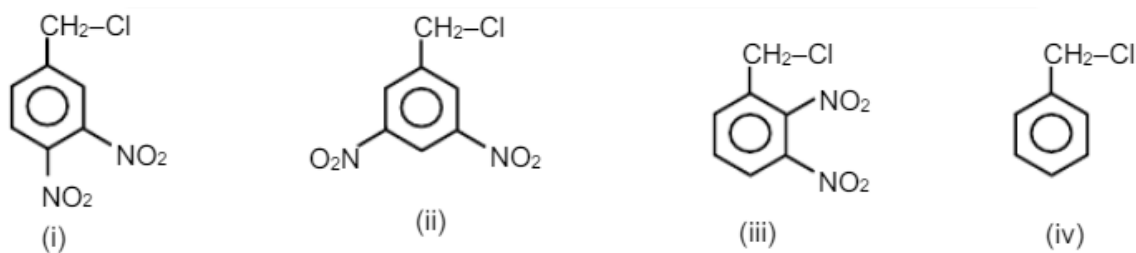
So molar mass of X = 100 g

$$\text{Molarity} = \frac{5}{100 \times 2} = 0.025M$$

Ans. = 0.025 M

M = 25 × 10–3

So P = 25

Question 34. Write down nucleophilic substitution (SN₂) order for following.

(i)   (ii)   (iii)   (iv)

(1) i > ii > iii > iv

(2) iv > iii > ii > i

(3) i > iii > ii > iv
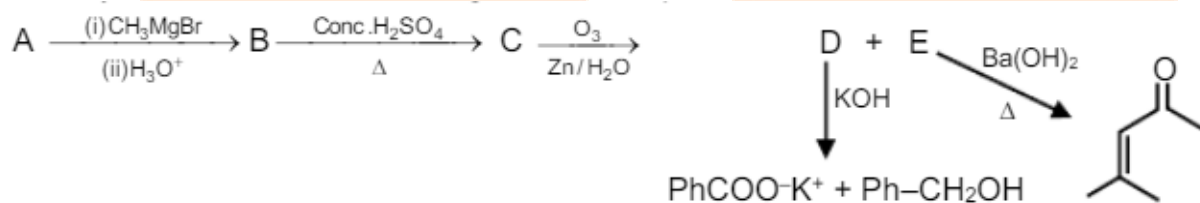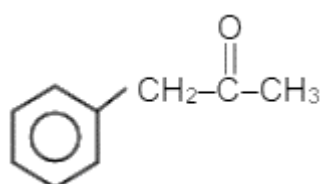
(4) iii > i > ii > iv

Answer: (3)

Solution:

SN₂ reaction depend upon – I, – M effect on substrate. On increase – I, – M, effect rate of SN₂ reaction increase.

Question 35. Identify structure of A in following reaction sequence.

(1)

(2)

(3)

(4)
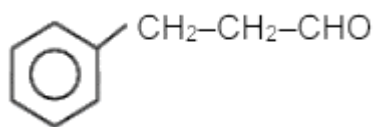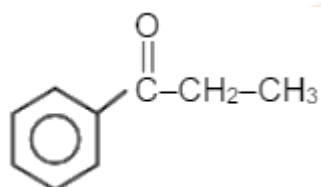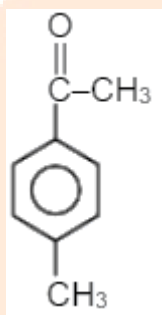
Answer: (1)

Solution:
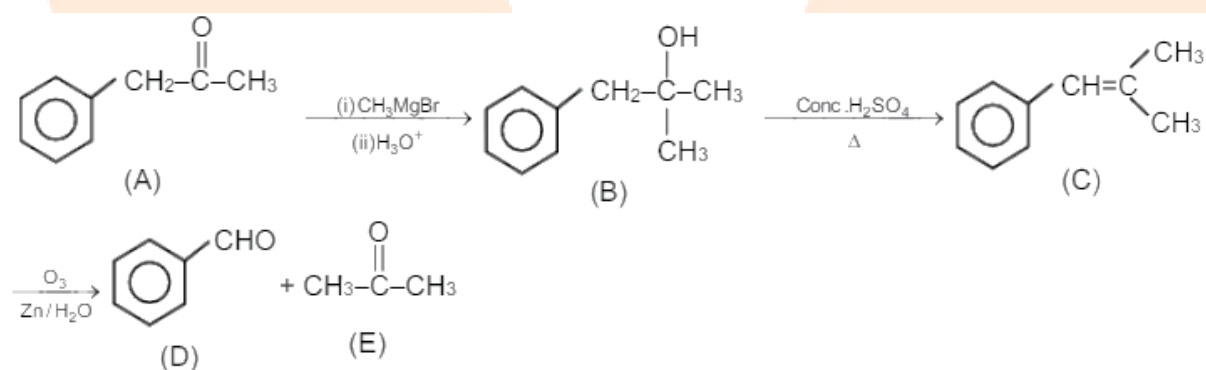
Question 36. Calculate $-\overset{O}{\underset{||}{C}}-$ in given structure of peptide chain.

Asp – Gly – Lys

Answer: 4.00

Solution:
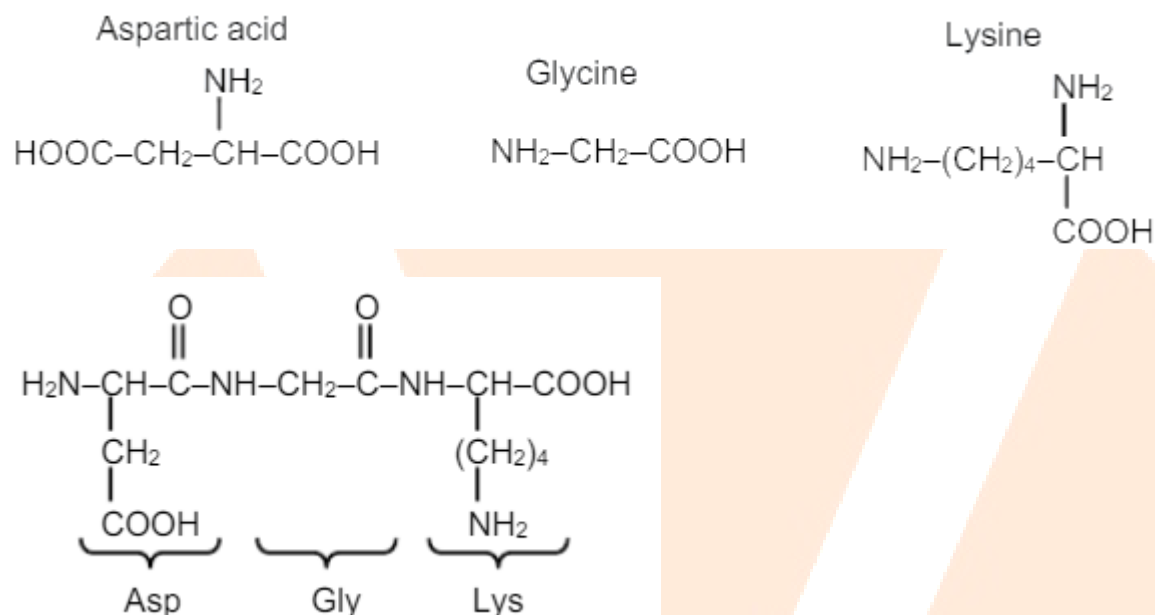
Question 37. Match the columns

Column–I	Column–II
(A) Chloramphenicol	(a) Antacid
(B) Ranitidine	(2) Antithistamine
(C) Phenelzine (nardil)	(3) Antibiotic
(D) Morphine	(d) Analgesic
	(5) Antidepressent

  (1) A–1; B–2; C–4; D–5

  (2) A–3; B–2; C–5; D–4

  (3) A–2; B–4; C–5; D–1

  (4) A–3; B–2; C–1; D–5

Answer: (2)

Solution:

Ranitidine → Antihistamines

Phenelzine (Nardil)

→ Antidepressant drugs

Chloramphenicol → Antibiotics

Morphine → Analgesics

Question 38.

$\xrightarrow[\Delta]{tBuO^-K}$
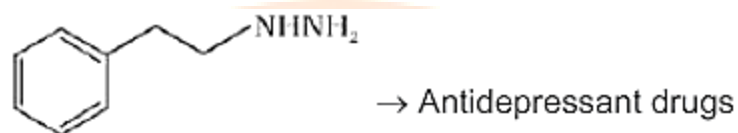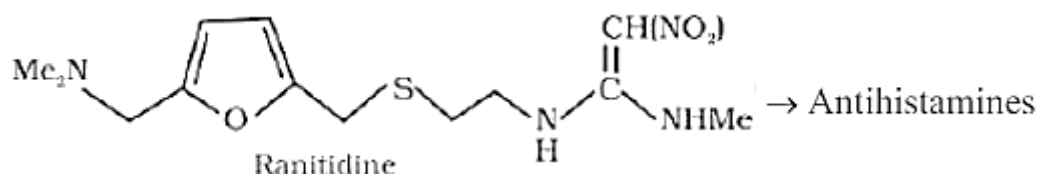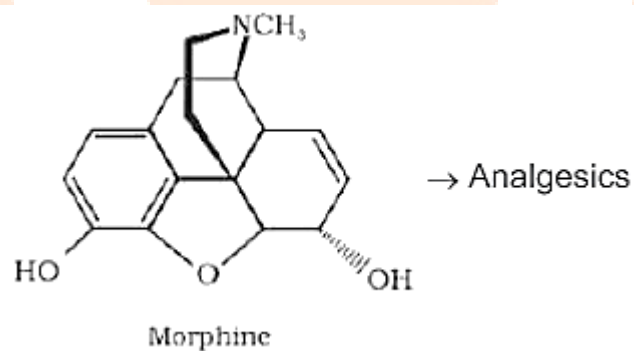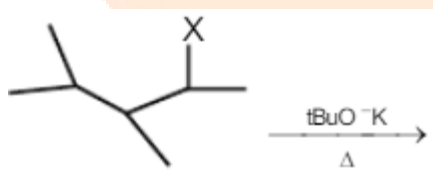
Find product of above reaction

(1)

(2)

(3)

(4)

Answer: (1)

Solution:

Major product
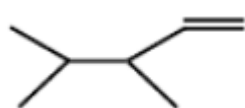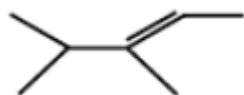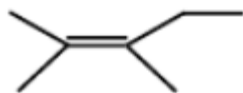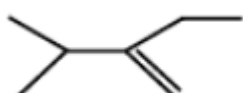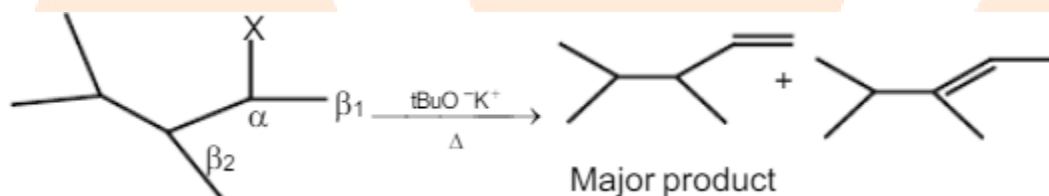
This is an example of E_2 reaction and due to bulky base final product is Hoffmann alkene.

Question 39.
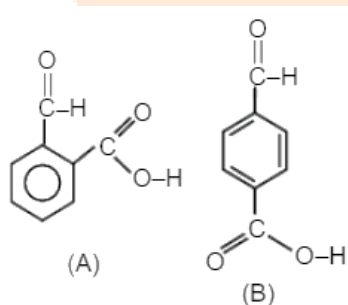
(A) B is less water soluble than A

(B) B is more crystalline in nature than A

(C) B has more boiling point than A

Select correct statement regarding above structures.

(1) A, B are correct

(2) B, C are correct

(3) Only C are correct

(4) A, B, C all are correct

Answer: (2)

Solution:

Due to inter molecular H-Bonding in B, than A, B is more soluble and having more B.P point than A.

Question 40. Write down decreasing order of Nucleophilic addition reaction of following Propanal, Butanone, Propanone, Benzaldehyde

(1) Propanal > Butanone > Propanone > Benzaldehyde

(2) Propanal > Benzaldehyde > Propanone > Butanone

(3) Propnone > Propanal > Butanone > Benzaldehyde

(4) Propnone > Butanone > Benzaldehyde > Propanal

Answer: (2)

Solution:

Rate of NAR $\alpha - 1 - M$ on substate
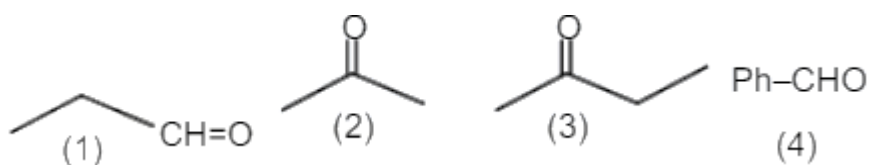
$1 > 4 > 2 > 3$

Question 41. Which of the following statement are incorrect statement(s) for acid rain

(A) It corrodes water pipes

(B) It is not harmful for trees and plants

(C) It does not cause breathing problem in human being and animals

(D) It damages building and other structures made of stone or metal.

 (1) A & B

 (2) B & C

 (3) A & C

 (4) B & D

Answer: (2)

Solution:

(B) It is harmful for trees and plants

(C) It causes breathing problem in human being and animals.

Mathematics

Question 42. If $\int_{0}^{\frac{1}{2}} \dfrac{x^2}{\left(1-x^2\right)^{3/2}} dx = \dfrac{k}{6}$, then k =

(1) $3\sqrt{2} + \pi$

(2) $2\sqrt{3} - \pi$

(3) $2\sqrt{3} + \pi$

(4) $3\sqrt{2} - \pi$

Ans. (2)

Sol. $\dfrac{k}{6} = \int\limits_{0}^{\frac{1}{2}} \dfrac{x^2}{\left(1-x^2\right)^{3/2}} dx \quad x = \sin\theta ; dx = \cos\theta\, d\theta$

$\Rightarrow \dfrac{k}{6} = \int\limits_{0}^{\frac{\pi}{6}} \dfrac{\sin^2\theta}{\left(1-\sin^2\theta\right)^{3/2}}.\cos\theta d\theta \quad \Rightarrow \dfrac{k}{6} = \int\limits_{0}^{\frac{\pi}{6}} \dfrac{\sin^2\theta}{\cos^3\theta}.\cos\theta d\theta$

$\Rightarrow \dfrac{k}{6} = \int\limits_{0}^{\frac{\pi}{6}} \tan^2\theta d\theta = \int\limits_{0}^{\frac{\pi}{6}}\left(\sec^2\theta - 1\right)d\theta \Rightarrow$

$\dfrac{k}{6} = \left(\tan\theta - \theta\right)_{0}^{\pi/6} = \left(\dfrac{1}{\sqrt{3}} - \dfrac{\pi}{6}\right) = \dfrac{2\sqrt{3} - \pi}{6}$

$\Rightarrow k = 2\sqrt{3} - \pi.$

Question 43. Let $\dfrac{x^2}{25} + \dfrac{y^2}{b^2} = 1$ and $\dfrac{x^2}{16} - \dfrac{y^2}{b^2} = 1$ are ellipse and hyperbola respectively such that $e_1 e_2 = 1$ where e_1 & e_2 are eccentricities. If distance between foci of ellipse is α and that of hyperbola is β then $(\alpha, \beta) =$

   (1) (4, 5)

   (2) (8, 10)

(3) (10, 7)

(4) (4, 10)

Ans. (2)

Sol. $e_1 = \sqrt{1 - \dfrac{b^2}{25}}; e_2 = \sqrt{1 + \dfrac{b^2}{16}}$

$e_1 e_2 = 1$

$\Rightarrow \left(e_1 e_2\right)^2 = 1 \qquad \Rightarrow \left(1 - \dfrac{b^2}{25}\right)\left(1 + \dfrac{b^2}{16}\right) = 1$

$\Rightarrow 1 + \dfrac{b^2}{16} - \dfrac{b^2}{25} - \dfrac{b^4}{25 \times 16} = 1$

$\Rightarrow \dfrac{9}{16.25} b^2 - \dfrac{b^2}{25.16} = 0 \Rightarrow b^2 = 9$

$e_1 = \sqrt{1 - \dfrac{9}{25}} = \dfrac{4}{5}$

$e_2 = \sqrt{1 + \dfrac{9}{16}} = \dfrac{5}{4}$

$\alpha = 2(5)(e_1) = 8$

$\beta = 2(4)(e_2) = 10$

$(\alpha, \beta) = (8, 10).$

Question 44. Two equal circles of radius $2\sqrt{5}$ passes through the extremities of Latus Rectum of $y2 = 4x$ then the distance between centres of circles is

    (1) 4

    (2) 8

    (3) 2

    (4) 6

Ans. (2)
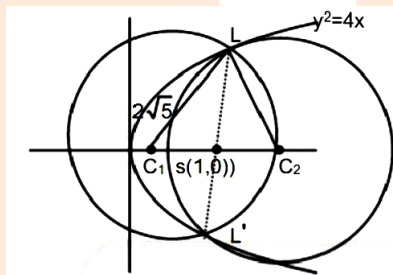
Sol.

$$C_1 C_2 = 2C_1 S = 2\sqrt{20-4} = 8$$

Question 45. If

$$\int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx = A(x)\tan^{-1}\sqrt{x} + B(x) + C,\text{ then A(x) and}$$

B(x) will be

    (1) $1+x, \sqrt{x}$

    (2) $1-x, -\sqrt{x}$

    (3) $1+x, -\sqrt{x}$

    (4) $1-x, \sqrt{x}$

Ans. (3)

Sol. $I = \int \sin^{-1}\left(\dfrac{\sqrt{x}}{\sqrt{1+x}}\right) dx$

Let, $x = t^2 \Rightarrow dx = 2dt$

$I = \int 2t \tan^{-1} t \, dt$

$= 2\left[\tan^{-1} t \cdot \dfrac{t^2}{2} - \int \dfrac{1}{1+t^2} \cdot \dfrac{t^2}{2} dt \right]$

$= 2\left[\tan^{-1} t \cdot \dfrac{t^2}{2} - \dfrac{1}{2} \int \dfrac{1+t^2-1}{1+t^2} dt \right]$

$= t^2 \cdot \tan^{-1} t - \int \left(1 - \dfrac{1}{1+t^2} \right) dt$

$= t^2 \tan^{-1} t - t + \tan^{-1} t + C$

$= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + C$

$= (x+1)\tan^{-1} \sqrt{x} - \sqrt{x} + C \Rightarrow (Ax) = x+1 \ \& \ B(x) = -\sqrt{x}$

Question 46. The coefficient of term independent of x in the expansion of $\left(\dfrac{3x^2}{2} - \dfrac{1}{3x}\right)^9$ is λ, then the value of 18λ is

    (1) 9

    (2) 7

    (3) 6

    (4) 4

Ans. (2)

Sol. $T_{r+1} = {}^9C_r \left(\dfrac{3x^2}{2}\right)^{9-r} \left(-\dfrac{1}{3x}\right)^r$

$= {}^9C_r \left(\dfrac{3}{2}\right)^{9-r} \left(-\dfrac{1}{3}\right)^r x^{18-3r}$

For the term independent of x put $18 - 3r = 0 \Rightarrow r = 6$

$\Rightarrow T_7 = {}^9C_6 \left(\dfrac{3}{2}\right)^3 \left(-\dfrac{1}{3}\right)^6 = {}^9C_3 \left(\dfrac{1}{6}\right)^3 = \dfrac{9 \times 8 \times 7}{3 \times 2 \times 1}\left(\dfrac{1}{6}\right)^3 = \left(\dfrac{7}{18}\right)$

Question 47. If $|z_1 - 1| = \mathrm{Re}(z_1), |z_2 - 1| = \mathrm{Re}(z_2)$ and arg $(z_1 - z_2) = \dfrac{\pi}{3}$, then $\mathrm{Im}(z_1 + z_2) =$

(1) $\dfrac{1}{\sqrt{3}}$

(2) $\dfrac{2}{\sqrt{3}}$

(3) $\dfrac{\sqrt{3}}{2}$

(4) $\sqrt{3}$

Ans. (2)

Sol. Let $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$

$$|z_1 - 1| = \text{Re}(z_1)$$

$$(x_1 - 1)^2 + y_1^2 = x_1^2$$

$$y_1^2 - 2x_1 + 1 = 0 \quad \ldots(1)$$

$$|z_2 - 1| = \text{Re}(z_2)$$

$$(x_2 - 1)^2 + y_2^2 = x_2^2$$

$$y_2^2 - 2x_2 + 1 = 0 \quad \ldots(2)$$

$$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$$

$$(y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2)$$

$$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right) \quad ..(3)$$

Also, $\arg(z_1 - z_2) = \dfrac{\pi}{3}$

$$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{3}$$

$$\frac{y_1 - y_2}{x_1 - x_2} = \sqrt{3} \quad \ldots(4)$$

$$\therefore \ y_1 + y_2 = \frac{2}{\sqrt{3}} \implies \text{Im}(z_1 + z_2) = \frac{2}{\sqrt{3}}.$$

Question 48. The probability of 5 digit numbers that are made up of exactly two distinct digit is

(1) $\dfrac{135}{10^4}$

(2) $\dfrac{125}{10^4}$

(3) $\dfrac{144}{10^4}$

(4) $\dfrac{127}{10^4}$

Ans. (1)

Sol. Total = 9(104)

Favourable = $^9C_2$ (25 - 2) + $^9C_1$ (24 − 1) = 36(30) + 9(15) = 1080 + 135

$$\text{Probability} = \frac{36 \times 30 + 9 \times 15}{9 \times 10^4} = \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}.$$

Question 49. Let $\left(\lambda^2 + 1\right)x^2 - 4\lambda x + 2 = 0$ be a quadratic equation, then set of values of λ if exactly one root of quadratic equation lies in (0, 1) is

(1) (2, 3)

(2) (1, 3)

(3) [1, 2)

(4) (1, 3]

Ans. (4)

Sol. $f(0).f(1) \leq 0$

$$\Rightarrow 2\left(\lambda^2 + 1 - 4\lambda + 2\right) \leq 0 \quad \Rightarrow 2\left(\lambda^2 - 4\lambda + 3\right) \leq 0$$

$$\left(\lambda - 1\right)\left(\lambda - 3\right) \leq 0$$

$$\Rightarrow \lambda \in \left[1, 3\right]$$

But at $\lambda = 1$, both roots are 1 so $\lambda \neq 1$

Question 50. The orthocentre of ΔABC where vertices are A(-1, 7), B(-7, 1), C(5, -5) is

    (1) (-3, 3)

    (2) (3, -3)

    (3) (3, 3)

    (4) (-3, -3)

Ans. (1)

Sol.
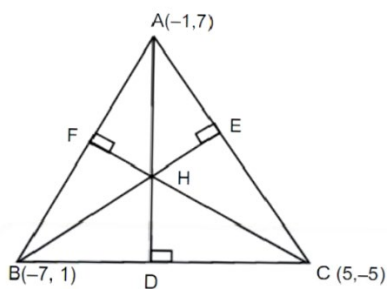
$$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$$

$\therefore$ Equation of AD is $y - 7 = 2(x + 1)$

$y = 2x + 9$ ...(1)

$$m_{AC} = \frac{12}{-6} = -2$$

$\therefore$ Equation of BE is

$$y - 1 = \frac{1}{2}(x + 7)$$

$$y = \frac{x}{2} + \frac{9}{2} \quad \text{...(2)}$$

By (1) and (2)

$$2x + 9 = \frac{x + 9}{2}$$

$\Rightarrow 4x + 18 = x + 9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

$\therefore y = 3$

Question 51. m A.M. and 3 GM are inserted between 3 and 243 such that 2nd Gm = 4th AM then m = .

Ans. (39)

Sol. 3, A1, A2, A3, ……. Am, 243

$$d = \frac{243 - 3}{m+1} = \frac{240}{m+1}$$

3, G1, G2, G3, 243

$$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$$

G2 = A4

$$\Rightarrow 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right) \Rightarrow 27 = 3 + \frac{960}{m+1} \Rightarrow m+1 = 40$$

$$\Rightarrow m = 39$$

Question 52. A normal is drawn to parabola y2 = 4x at (1, 2) and tangent is drawn to y = ex at (c, ec). If tangent and normal intersect at x-axis then find C.

Ans. 04.00

Sol. For (1, 2) of y2 = 4x $\Rightarrow$ t = 1, a = 1

   Normal $\Rightarrow$ tx + y = 2at + at3

$\Rightarrow$ x + y = 3 intersect x-axis at (3, 0)

$$y = e^x \Rightarrow \frac{dy}{dx} = e^x$$

Tangent $\Rightarrow y - e^c = e^c (x - c)$

At (3, 0)

$$\Rightarrow 0 - e^c = e^c (3 - c) \Rightarrow c = 4$$

Question 53. If relation $R_1 = \{(a,b) : a, b \in R, a^2 + b^2 \in Q\}$

and $R_2 = \{(a,b) : a, b \in R, a^2 + b^2 \notin Q\}$

Then

    (1) R_1 is transitive, R_2 is not transitive

    (2) R_1 is not transitive, R_2 is not transitive

    (3) R_1 is transitive, R_2 is transitive

    (4) R_1 is not transitive, R_2 is transitive

Ans. (2)

Sol. For R_1, let $a = 1 + \sqrt{2}, b = 1 - \sqrt{2}, c = 8^{1/4}$

$aR_1b \Rightarrow a^2 + b^2 = \left(1 + \sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 6 \in Q$

$bR_1c \Rightarrow b^2 + c^2 = \left(1 - \sqrt{2}\right)^2 + \left(8^{1/4}\right)^2 = 3 \in Q$

$aR_1c \Rightarrow a^2 + c^2 = \left(1 + \sqrt{2}\right)^2 + \left(8^{1/4}\right)^2 = 3 + 4\sqrt{2} \notin Q$

$\therefore$ R_1 is not transitive.

For R_2, let $a = 1 + \sqrt{2}, b = \sqrt{2}, c = 1 - \sqrt{2}$

$aR_2b \Rightarrow a^2 + b^2 = \left(1 + \sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 = 5 + 2\sqrt{2} \notin Q$

$bR_2c \Rightarrow b^2 + c^2 = \left(\sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 5 - 2\sqrt{2} \notin Q$

$aR_2c \Rightarrow a^2 + c^2 = \left(1 + \sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 6 \in Q$

$\therefore$ R_2 is not transitive

Question 54. If the sum of first n terms of series

$$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots \text{is } 488 \text{ and nth term is negative}$$

then find n

    (1) -4

    (2) 4

    (3) 1

    (4) 6

Ans. (1)

Sol. Given series is: $\dfrac{100}{5} + \dfrac{98}{5} + \dfrac{96}{5} + \dfrac{94}{5} + \ldots\ldots$

    This forms an AP with $a = \dfrac{100}{5}$, $d = -\dfrac{2}{5}$

    $T_n < 0$

$$\Rightarrow \frac{100}{5} + (n-1)\left(\frac{-2}{5}\right) < 0$$

$$\Rightarrow n > 51$$

$$488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(-\frac{2}{5}\right)\right]$$

$$\Rightarrow n^2 - 101n + 2440 = 0$$

$$\Rightarrow n = 61 \text{ or } 40$$

Since n > 51, therefore n = 61

$$T_n = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$$

Question 55. Surface area of cube is increasing at rate of 3.6 cm2/s. Find the rate at which its volume increases when length of side a is 10 cm.

    (1) 9

    (2) 10

    (3) 18

    (4) 20

Ans. (1)

Sol.

$$S = 6a^2 \Rightarrow \frac{dS}{dt} = 12a.\frac{da}{dt} = 3.6 \Rightarrow 12(10)\frac{da}{dt} = 3.6 \Rightarrow \frac{da}{dt} = 0.03$$

$$V = a^3 \Rightarrow \frac{dV}{dt} = 3(10)^2.\left(\frac{3}{100}\right) = 9\,\text{cm}^3/\text{s}$$

Question 56. Which of the following point lies on plane containing lines

$$\bar{r} = \hat{i} + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) \text{and } \vec{r}- = -\hat{j} + \mu\left(-\hat{i} - 2\hat{j} + \hat{k}\right).$$

    (1) (1, 3, 6)

(2) (1, -3, 6)

(3) (-2, 1, 2)

(4) (1, 3, 1)

Ans. (2)

Sol. Normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & -2 & 1 \end{vmatrix}$

$\bar{n} = 3\hat{i} - 2\hat{j} - \hat{k}$

D.R. 's = 3, -2, -1

Plane $\Rightarrow 3(x - 1) - 2(y - 0) - 1(z - 0) = 0$

$\Rightarrow 3x - 2y - z - 3 = 0$

Question 57. $\lim\limits_{x \to a} \dfrac{\left(a^2 + 2x^2\right)^{1/3} - \left(3x^2\right)^{1/3}}{\left(3a^2 + x^2\right)^{1/3} - \left(4x^2\right)^{1/3}} =$

(1) $\left(\dfrac{4}{3}\right)^{2/3}$

(2) $\dfrac{1}{3}\left(\dfrac{3}{4}\right)^{2/3}$

(3) $\dfrac{1}{3}\left(\dfrac{2}{3}\right)^{2/3}$

(4) $\dfrac{1}{3}\left(\dfrac{4}{3}\right)^{2/3}$

Ans. (4)

Sol. $\displaystyle\lim_{x\to a}\left(\dfrac{\dfrac{1}{3}\left(a^2+2x^2\right)^{-2/3}.4x-\dfrac{1}{3}.\left(3x^2\right)^{-2/3}.6x}{\dfrac{1}{3}\left(3a^2+x^2\right)^{-2/3}.2x-\dfrac{1}{3}\left(4x^2\right)^{2/3}.8x}\right)$

$=\dfrac{\dfrac{1}{3}\left(3a^2\right)^{-2/3}.a(4-6)}{\dfrac{1}{3}\left(4a^2\right)^{-2/3},a(2-8)}=\dfrac{3^{\frac{-2}{3}}}{4^{\frac{-2}{3}}}.\dfrac{1}{3}$

$=\dfrac{4^{\frac{2}{3}}}{3^{\frac{2}{3}}}.\dfrac{1}{3}=\left(\dfrac{4}{3}\right)^{\frac{2}{3}}.\dfrac{1}{3}$

Question 58. If $x^3dy + xydx = 2ydx + x^2dy$ and $y(2) = e$ then $y(4) = ?$

(1) $\dfrac{1}{2}+\sqrt{e}$

(2) $\dfrac{1}{2}\sqrt{e}$

(3) $\sqrt{e}$

(4) $\dfrac{3}{2}\sqrt{e}$

Ans. (4)

Sol. $x^3dy + xydx + 2ydx + x^2dy$

$$\Rightarrow \left(x^3 - x^2\right)dy = \left(2 - x\right)ydx$$

$$\Rightarrow \frac{dy}{y} = \frac{2-x}{x^2\left(x-1\right)}dx$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{2-x}{x^2\left(x-1\right)}dx \; ...(i)$$

Let $\dfrac{2-x}{x^2\left(x-1\right)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1}$

$\Rightarrow 2 - x = Ax(x-1) + B(x-1) + Cx2$

$\Rightarrow C = 1, B = -2$ and $A = -1$

$$\Rightarrow \int \frac{dy}{y} = \int \left\{ \frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-1} \right\}dx$$

$$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln\left|x-1\right| + C$$

$\therefore\; y(2) = e$

$\Rightarrow 1 = $ -ln2 + 1 + 0 + C

$\Rightarrow C = \ln 2$

$\ln y = -\ln x + \dfrac{2}{x} + \ln|x-1| + \ln 2$

at $x = 4$

$$\Rightarrow \ln y\left(4\right) = -\ln 4 + \frac{1}{2} + \ln 3 + \ln 2$$

$$\Rightarrow \ln y(4) = \ln\left(\frac{3}{2}\right) + \frac{1}{2} = \ln\left(\frac{3}{2}e^{1/2}\right)$$

$$\Rightarrow y(4) = \frac{3}{2}e^{1/2}$$

Question 59. Find the number of 3 digit numbers if sum of their digits is 10

Ans. (55.00)

Sol. Let xyz be the three digit number

$$x + y + z = 10, \ x \leq 1, \ y \geq 0, \ z \geq 0$$

$$x - 1 = t \Rightarrow \ x = 1 + t \ x - 1 \geq 0$$

$$t \geq 0$$

$$t + y + z = 10 - 1$$

$$t + y + z = 9, \ 0 \leq t, z, z \leq 9$$

$\therefore$ Non-negative integral solutions $= {}^{9+3-1}C_{3-1}$

$$= {}^{11}C_2 = \frac{11.10}{2} = 55$$

Question 60. If $\dfrac{a}{\cos\theta} = \dfrac{b}{\cos\left(\theta + \dfrac{2\pi}{3}\right)} = \dfrac{c}{\cos\left(\theta + \dfrac{4\pi}{3}\right)}$ then

find angle between vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ if

$\theta = \dfrac{2\pi}{9}$ and $a^2 + b^2 + c^2 = 1$, is

(1) $\dfrac{\pi}{3}$

(2) $\dfrac{\pi}{6}$

(3) $\dfrac{2\pi}{3}$

(4) $\dfrac{5\pi}{6}$

Ans. (3)

Sol.

$$\dfrac{a}{\cos\theta} = \dfrac{b}{\cos\left(\theta + \dfrac{2\pi}{3}\right)} = \dfrac{c}{\cos\left(\theta + \dfrac{4\pi}{3}\right)}$$

$$= \dfrac{a + b + c}{\cos\theta + \cos\left(\theta + \dfrac{2\pi}{3}\right) + \cos\left(\theta + \dfrac{4\pi}{3}\right)} = \dfrac{a + b + c}{0}$$

$$\Rightarrow a + b + c = 0 \Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$$

$$\Rightarrow ab + bc + ca = -\frac{1}{2}$$

Now let angle between given vectors is f

$$\therefore \cos\phi = \frac{\left(a\hat{i} + b\hat{j} + c\hat{k}\right).\left(b\hat{i} + c\hat{j} + a\hat{k}\right)}{a^2 + b^2 + c^2}$$

$$\cos\phi = \frac{ab + bc + ca}{1} = \frac{-1}{2}$$

$$\phi = \frac{2\pi}{3}$$

Question 61. If $(p \wedge q) \rightarrow (\sim q \vee r)$ has truth value false then the truth values of p, q, r respectively are

    (1) T, T, F

    (2) T, F, T

    (3) F, F, T

    (4) T, T, T

Ans. (1)

Sol. $(p \wedge q)$ should be TRUE and $(\sim q \vee r)$ should be FALSE.